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About the non-existence of small breathers in Klein-Gordon equations

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Resumen

Breathers are solutions of evolutionary PDEs, which are periodic in time and spatially localized. They are known to exist for the sine-Gordon equation but are believed to be rare in other Klein-Gordon equations. When the spatial dimension is equal to one, exchanging the roles of time and space variables (in the so-called spatial dynamics framework), breathers can be interpreted as homoclinic solutions to steady solutions (in an infinite dimensional space of periodic in time solutions). Thus arise from the intersections of the stable and unstable manifolds of the steady states.

In this talk, we shall study small breathers of the nonlinear Klein-Gordon equation generated in an unfolding bifurcation as a pair of eigenvalues collide at the origin when a parameter (temporal frequency) varies. Due to the presence of the oscillatory modes, generally the finite dimensional stable and unstable manifolds do not intersect in the infinite dimensional phase space, but with an exponentially small splitting (relative to the amplitude of the breather) in this singular perturbation problem of multiple time scales.

We will explain how to obtain an asymptotic formula for the distance between the stable and unstable manifold of the steady solutions when the steady solution has weakly hyperbolic one dimensional stable and unstable manifolds. This formula allows to say that for a wide set of Klein-Gordon equations breathers do not exist.

Due to the exponential small splitting, classical perturbative techniques cannot be applied to this problem.

Referencias

[1] O.M.L. Gomide, M. Guardia, T. M-Seara, Ch. Zeng. *On small breathers of nonlinear Klein-Gordon equations via exponentially small homoclinic splitting.* Preprint at archiv: *https : //arxiv.org/pdf/*2107,14566.*pdf*